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Particle gravitation theories of the Hoyle–Narlikar type

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Abstract. In the Hoyle–Narlikar particle theory of gravitation, the mass function at a point X due to a particle a is defined as $m^{(a)}(X) = -\lambda\int\tilde{G}(X, A)\mathrm{d}a$ where $\tilde{G}(X, A)$ is a Green's function which satisfies a simple wave equation. In this paper the coupling constant λ is replaced by λ_a where λ_a can be positive or negative. Two possible subsequent generalizations of the action of the Hoyle–Narlikar theory are suggested and the two resultant particle theories developed. One special case of one of these theories is examined in some detail. This has particles which contribute negative inertia as well as those which contribute positive inertia and all the λ_a such that $|\lambda_a| = \lambda$. In this theory the gravitational 'constant' G is a function of position and can be positive or negative.

1. Introduction

In the particle theory of gravitation proposed by Hoyle and Narlikar (to be referred to as HN) (1964b, 1966), the proper time a of the particle a is given by

$$\mathrm{d}a^2 = g_{i_A j_A}\,\mathrm{d}x^{i_A}\,\mathrm{d}x^{j_A} \tag{1}$$

where A is a typical point on the world line of a and has coordinates $a^{i_A}(i_A = 0, 1, 2, 3)$. The g_{ij} are components of the metric tensor of the Riemannian space in which the world lines are embedded. The mass function at a general point X due to a is defined as

$$m^{(a)}(X) = -\lambda\int\tilde{G}(X, A)\,\mathrm{d}a \tag{2}$$

where λ is a positive coupling constant and $\tilde{G}(X, A)$ is a symmetric Green's function which satisfies the wave equation

$$\Box\tilde{G}(X, A)+\tfrac{1}{6}R\tilde{G}(X, A) = -\overset{4}{\delta}_{(X,A)}(-\bar{g})^{-1/2} \tag{3}$$

$\bar{g}$ being the determinant of the parallel propagators $\bar{g}_{i_A j_A}$. The mass m_a of the particle a arises from all the other particles of the universe and is given by

$$m_a(A) = \sum_{b\neq a} m^{(b)}(A) = -\lambda\sum_{b\neq a}\int\tilde{G}(A, B)\,\mathrm{d}b. \tag{4}$$

The physical significance of the mass field is that $m^{(a)}(X)$ is the intertia contributed by a particle a to another at the point X. The inertial mass of a particle is equal to the inertia contributed to it by all the other particles.

The action assumed by HN has the form

$$J = -\sum_a\tfrac{1}{2}\int m_a\,\mathrm{d}a = \lambda\sum_{a<b}\sum\int\int\tilde{G}(A, B)\,\mathrm{d}a\,\mathrm{d}b. \tag{5}$$

The factor $\frac{1}{2}$ arises because each $\tilde{G}(A, B)$ is shared by two particles a and b. The particle equations formed by varying J with respect to g_{ij} and putting $\delta J = 0$ are

$$\sum_a \sum_b H(a, b)_{ij} \equiv \sum_{a<b}\sum \{m^{(a)}m^{(b)}(R_{ij} - \tfrac{1}{2}g_{ij}R) + m^{(a)}(m^{(b)}{}_{;ij} - g_{ij}\Box m^{(b)}) + m^{(b)}(m^{(a)}{}_{;ij} - g_{ij}\Box m^{(a)}) - 2(m^{(a)}{}_{;i}m^{(b)}{}_{;j} + m^{(a)}{}_{;j}m^{(b)}{}_{;i} - \tfrac{1}{2}g_{ij}m^{(a);k}m^{(a)}{}_{;k})\} = -3\lambda T_{ij} \tag{6}$$

where

$$T^{ij}(X) = \sum_a \int m_a \delta^4{}_{(X,A)}\{-\bar{g}(X, A)\}^{-1/2} \frac{\mathrm{d}a^{i_A}}{\mathrm{d}a}\frac{\mathrm{d}a^{j_A}}{\mathrm{d}a} \bar{g}_{i_A}{}^{i}\bar{g}_{j_A}{}^{j}\,\mathrm{d}a. \tag{7}$$

These equations are conformally invariant under the mapping

$$\begin{aligned} g^*_{ij} &= \Omega^2 g_{ij} & m^{*(a)} &= \frac{m^{(a)}}{\Omega} \\ \tilde{G}^* &= \frac{\tilde{G}}{\Omega^2} & \mathrm{d}a^* &= \Omega\,\mathrm{d}a. \end{aligned} \tag{8}$$

The terms on the left side of (6) involving derivatives of $m^{(a)}$ have no equivalent in the usual form of the field equations of Einstein whilst the other terms obviously do. They do, however, have equivalent terms in the conformally invariant version of Einstein's equations, i.e. equations (29–31). The portion of (6) equivalent to the gravitational constant G in Einstein's equation is

$$G = \alpha\lambda \left\{ \sum_{a<b}\sum m^{(a)}m^{(b)} \right\}^{-1} \qquad \alpha = \text{constant} > 0 \tag{9}$$

and it is a function of position and time.

Deser and Pirani (1965, 1967) comment on the sign of G in the HN theory and point out that it could be changed by replacing the coefficient $\frac{1}{6}$ in the wave equation (3) by a negative number although this cannot be done if the theory is to be kept conformally invariant under (8). They then discuss the sign of G in the Einstein theory. In this paper, however, the restriction of conformal invariance will be kept in general.

Islam (1967) states that his work implies that the sign in the definition (2) of $m^{(a)}(X)$ is correctly chosen. A change of sign does not affect the geometry or geodesics of particles but makes the sign of G, given by (9), negative. However the inertial mass m_a of particle a and the inertia contributed by it are both now negative. Thus a positive (or negative) G means that the masses are positive (or negative) though in Einstein's field theory the possibility arises of G being positive whilst the masses are negative and vice versa. In this last case, however, the geometry is different from the usual one.

Hawking (1965) discusses briefly the action

$$J = \sum_{a<b}\sum q_a q_b \iint \tilde{G}(A, B)\,\mathrm{d}a\,\mathrm{d}b \qquad q_a, q_b = \pm 1 \tag{10}$$

where q_a and q_b are gravitational charges analogous to electric charges. He requires that there are the same number of positive and negative particles and suggests that the resultant theory is an alternate to the HN theory. However he does not extend the discussion to include any particle equations.

HN (1967) suggest that their action could be written as

$$J = \sum_{a<b}\sum \int\int w_a w_b \tilde{G}(A, B)\, \mathrm{d}a\, \mathrm{d}b \tag{11}$$

where w_a is a numerical weight factor associated with particle a such that particles with different masses could be included in the theory. They make no suggestion that the w_a could be negative as well as positive and, for simplicity, work with (5).

The author has suggested an extension of the HN theory such that mass functions of two types are included in the theory (McIntosh 1970 to be referred to as I). Particles of one type have a negative sign in (2) and contribute positive inertia, and those of the other type have a positive sign in (2) and contribute negative inertia. In this paper the mass function (2) is generalized so that particles can contribute inertias with different signs and magnitudes. Two different possible subsequent generalizations of the action J are suggested such that two possible theories result. One special case of one of the theories is examined in some detail.

2. Basic equations

Instead of (2), define the mass function at the point X due to particle a by

$$m^{(a)}(X) = -\lambda_a \int \tilde{G}(X, A)\, \mathrm{d}a \tag{12}$$

where $\tilde{G}(X, A)$ satisfies the wave equation (3). Here λ_a is some constant scalar factor associated with particle a and can be either positive or negative such that $m^{(a)}$ is either positive or negative. Then define

$$m_a(A) = \sum_{b \neq a} m^{(b)}(A) \tag{13}$$

such that m_a arises from the mass functions of all the other particles in the universe. This is the same as (4) except that it can be positive, negative, or zero. The sign of (13) depends upon the position and magnitude of the other particles.

There are now two obvious ways of redefining the action J. These will be called J_{I} and J_{II} where

$$J_{\mathrm{I}} = -\tfrac{1}{2}\sum_a \int m_a\, \mathrm{d}a \tag{14–I}$$

and

$$J_{\mathrm{II}} = -\tfrac{1}{2}\sum_a \int \lambda_a m_a\, \mathrm{d}a. \tag{14–II}$$

From (12)–(14)

$$J_{\mathrm{I}} = \sum_{a<b}\sum \tfrac{1}{2}(\lambda_a + \lambda_b) \int\int \tilde{G}(A, B)\, \mathrm{d}a\, \mathrm{d}b \tag{15–I}$$

which can easily be seen by writing out a number of terms of the series, and

$$J_{\mathrm{II}} = \sum_{a<b}\sum \lambda_a \lambda_b \int\int \tilde{G}(A, B)\, \mathrm{d}a\, \mathrm{d}b. \tag{15–II}$$

This last equation is the same as (10) with $\lambda_a = q_a$ or (11) with $\lambda_a = w_a$.

When variation of these actions with respect to g_{ij} is carried out and $\delta J = 0$, two sets of particle equations are formed and two particle theories, labelled I and II, result. The variations follow the same lines as in the HN theory and thus the working will not be repeated. Such variation gives from J_{I} the particle equations I:

$$\sum_{a<b}\sum \tfrac{1}{2}\left(\frac{1}{\lambda_a} + \frac{1}{\lambda_b}\right) H(a, b)_{ij} = -3T_{ij} \tag{16–I}$$

and for J_{II} the particle equations II:

$$\sum_{a<b}\sum H(a, b)_{ij} = -3T(\lambda)_{ij} \tag{16–II}$$

$H(a, b)_{ij}$ being defined by (6), T_{ij} by (7) and

$$T(\lambda)^{ij} = \sum_a \int \lambda_a m_a \delta^4[-\bar{g}]^{-1/2} \frac{\mathrm{d}a^i}{\mathrm{d}a}\frac{\mathrm{d}a^j}{\mathrm{d}a}\,\mathrm{d}a \tag{17}$$

where obvious abbreviations have been made. Both of these theories are conformally invariant under (8).

If all the λ_a are equal to λ, both of these theories revert to the original HN theory.

Direct particle electrodynamic and C-field terms could be added to either theory as with the HN theory though the addition of C-field terms would mean that the conformal invariance is lost.

3. Particle theories I and IA

In the particle theory I with action J_{I}, m_a of (13) is the mass of the particle a. G in (9) is replaced by

$$G = \alpha\left\{\sum_{a<b}\sum \tfrac{1}{2}\left(\frac{1}{\lambda_a} + \frac{1}{\lambda_b}\right) m^{(a)} m^{(b)}\right\}^{-1}. \tag{18}$$

The factor $\frac{1}{2}(1/\lambda_a + 1/\lambda_b)$ arises in the calculations as $\frac{1}{2}(\lambda_a + \lambda_b)/\lambda_a\lambda_b$. $T^{ij}(X)$ can be positive, zero or negative at X.

The following special case of theory I will be called theory IA. This is the case where there are both positive and negative $m^{(a)}$ and where all the λ_a have the same magnitude, λ. Let the a for which $m^{(a)}$ is positive (i.e. λ_a positive) have a subscript 1 and let the a for which $m^{(a)}$ is negative (i.e. λ_a negative) have a subscript 2. Then

$$m^{(a_\nu)}(X) = (-1)^\nu \lambda \int \tilde{G}(X, A_\nu)\,\mathrm{d}a_\nu \qquad \nu = 1, 2 \tag{19}$$

where there is no summation over the νs. Here λ has replaced $-\lambda_a$ whenever λ_a is negative, that is

$$|\lambda_a| = \lambda \qquad \text{for all } a. \tag{20}$$

The particle equations (16–I) then become

$$\sum_{\nu=1,2} (-1)^{\nu+1} \sum_{a_\nu < b_\nu}\sum H(a_\nu, b_\nu)_{ij} = -3\lambda T_{ij} \tag{21}$$

there

$$T^{ij} = \sum_{\nu=1,2} \sum_{a_\nu} \int m_{a_\nu} \delta^4[-\bar{g}]^{-1/2} \frac{\mathrm{d}a_\nu{}^i}{\mathrm{d}a}\frac{\mathrm{d}a_\nu{}^j}{\mathrm{d}a}\,\mathrm{d}a \tag{22}$$

which is the same as (7) since

$$\sum_a m_a = \sum_{a_1} m_{a_1} + \sum_{a_2} m_{a_2}.$$

Equation (21) is the same as (1) of paper I with $\lambda_1 = \lambda_2 = \lambda$.

The effect of having masses which contribute either positive or negative inertia on the equations (21) compared with (6) is to reduce the magnitude of the terms. The left hand side of (21) is the difference between two positive terms and the right hand side is the sum of a number of positive and a number of negative terms, though which individual term is positive and which is negative is not apparent since, for example, each m_{a_1} is not necessarily positive. Notice that there are no cross product terms of a_1 and a_2 type particles in the left side of (21). Also

$$G = \alpha\lambda\left\{\sum_{a_1 < b_1}\sum m^{(a_1)}m^{(b_1)} - \sum_{a_2 < b_2}\sum m^{(a_2)}m^{(b_2)}\right\}^{-1} \tag{23}$$

and can be positive or negative in different regions of space–time.

4. Particle theory II

In the particle theory II, the mass of the particle a is defined as

$$\lambda_a m_a(A) = \lambda_a \sum_{b \neq a} m^{(b)}(A) \tag{24}$$

and can be positive, negative, or even zero. λ_a here is HN's numerical weight factor w_a or Hawking's charge q_a. The action J_{II} of (15–II) is analogous to the action of the electrodynamics theory of HN (1964a, 1967).

If as above there are particles a_1 contributing positive inertia and particles a_2 contributing negative inertia, then (16–II) has the form

$$\sum_{a_1 < b_1}\sum H(a_1, b_1)_{ij} + \sum_{a_2 < b_2}\sum H(a_2, b_2)_{ij} - \tfrac{1}{2}\left|\sum_{a_1 < a_2}\sum H(a_1, a_2)_{ij}\right| = -3T(\lambda)_{ij} \tag{25}$$

such that the left side can still be negative. Indeed if there are N_1 particles of type a_1 and N_2 of type a_2, then there are $\tfrac{1}{2}N_1(N_1-1)$, $\tfrac{1}{2}N_2(N_2-1)$ and N_1N_2 terms in the three summations on the left side. The λ_a appear explicitly in $T(\lambda)_{ij}$ but appear implicitly in the $H(a, b)_{ij}$ terms in that they help determine the $m^{(a)}$.

G here is

$$G = \alpha\left\{\sum_{a<b}\sum m^{(a)}m^{(b)}\right\}^{-1} \tag{26}$$

and can be positive or negative.

5. Smooth fluid approximation

HN make a smooth fluid approximation of their theory by writing

$$m(X) = \sum_a m^{(a)}(X) \tag{27}$$

$$\tfrac{1}{2}m^2 \simeq \sum_{a<b}\sum m^{(a)}m^{(b)} \tag{28}$$

i.e. terms of the type $(m^{(a)})^2$ are neglected in comparison with those of the type $m^{(a)}m^{(b)}$, $b \neq a$. These terms may be regarded as 'self-action' terms. On a macroscopic scale, with the number of particles very large, the approximation is very good and the particle theory and the resultant field theory give very close predictions. The approximation cannot be made with any accuracy on a microscopic scale and on this scale the two theories have vastly different predictions. Under the approximation the particle equations (6) become the field equations

$$H_{ij} = -3\lambda T_{ij} \tag{29}$$

where

$$H_{ij} \equiv \tfrac{1}{2}\{m^2(R_{ij}-\tfrac{1}{2}g_{ij}R)+2m(m_{;ij}-g_{ij}\Box m)-4(m_{;i}m_{;j}-\tfrac{1}{4}g_{ij}m_{;k}m^{;k})\} \tag{30}$$

and

$$T^{ij} \simeq m\sum_a \int \delta^4[-g]^{-1/2}\frac{\mathrm{d}a^i}{\mathrm{d}a}\frac{\mathrm{d}a^j}{\mathrm{d}a}\,\mathrm{d}a. \tag{31}$$

These field equations thus do not hold in the neighbourhood of the particles.

In the particle theory I, however, to obtain field equations from the particle equations (16–I), (28) would have to be replaced by a relationship of the type

$$\frac{m^2}{\lambda} \simeq \sum_{a<b}\sum m^{(a)}m^{(b)}\left(\frac{1}{\lambda_a}+\frac{1}{\lambda_b}\right) \tag{32}$$

in which case the field equations (29) again result. This does not appear very satisfactory.

The theory IA, however, does have an acceptable form in the smooth fluid case. Here define

$$m_\nu = \sum_{a_\nu} m^{(a\nu)} \qquad \nu = 1,2 \tag{33}$$

$$\tfrac{1}{2}m_\nu{}^2 \simeq \sum_{a_\nu<b_\nu}\sum m^{(a\nu)}m^{(b\nu)} \qquad \nu = 1,2 \tag{34}$$

where m_1 is positive and m_2 is negative. (21) then becomes

$$(H_{1ij}-H_{2ij}) = -3\lambda(T_{1ij}-T_{2ij}) = -3\lambda T_{ij} \tag{35}$$

where $H_{\nu ij}$ and $T_{\nu ij}$ are (30) and (31) with m and a replaced by m_ν and a_ν respectively. This is the same as equation (2) of paper I with $\lambda_1 = \lambda_2 = \lambda$. The particles are thus being approximated to by two fluids, one for the a_1-type particles and another for the a_2-type particles.

In theory II under the smooth fluid approximation, the particle equations (16–II) become the field equations

$$H_{ij} = -3T(\lambda)_{ij} \tag{36}$$

where

$$T(\lambda)^{ij} \simeq m\sum_a \lambda_a \int \delta^4[-g]^{-1/2}\frac{\mathrm{d}a^i}{\mathrm{d}a}\frac{\mathrm{d}a^j}{\mathrm{d}a}\,\mathrm{d}a. \tag{37}$$

6. The field equations for the theory IA

The field equations (35) for the theory IA are

$$\sum_{\nu=1,2}(-1)^{\nu+1}\{m_\nu{}^2(R_{ij}-\tfrac{1}{2}g_{ij}R)+2m_\nu(m_{\nu;ij}-g_{ij}\Box m_\nu)-4(m_{\nu;i}m_{\nu;j}-\tfrac{1}{4}g_{ij}m_{\nu;k}m^{;k})\}$$
$$= -6\lambda T_{ij}. \tag{38}$$

The gravitational 'constant' G is

$$G = \alpha(m_1^2 - m_2^2)^{-1} \qquad \alpha = \text{constant} > 0 \tag{39}$$

such that

$$G > (<)0 \qquad \text{for } m_1^2 > (<)m_2^2. \tag{40}$$

As a hypersurface with $m_1^2 - m_2^2 = 0$ is approached from the region with $G > (<)\ 0$, $G \to +(-)\infty$. On such a hypersurface the left side of (38) is zero and therefore T_{ij} must be zero. When the covariant derivative of (38) is taken and the identities

$$R^{ij}m_{\nu;j} = (m_\nu{}^{;ij} - g^{ij}\Box m_\nu)_{;j} \tag{41}$$

are used, it follows that

$$\lambda T^{ij}{}_{;j} = \{(m_1 m_1{}^{;i} - m_2 m_2{}^{;i})\lambda T + (m_1 m_2{}^{;i} - m_2 m_1{}^{;i})(m_2\Box m_1 - m_1\Box m_2)\} \times (m_1^2 - m_2^2)^{-1}. \tag{42}$$

These equations are conformally invariant under the mapping

$$g_{ij}{}^* = \Omega^2 g_{ij} \qquad m_\nu{}^* = m_\nu/\Omega. \tag{43}$$

If treated just as a field theory, (38) could have been derived from the action

$$J = \int \sum_{\nu=1,2} (-1)^\nu \{m_{\nu;k}m_\nu{}^{;k} - \tfrac{1}{6}Rm_\nu{}^2\}(-g)^{1/2}\,\mathrm{d}^4x + 2\lambda \sum_a m_a \int \mathrm{d}a. \tag{44}$$

7. Special conformal frames for theory IA with G constant

HN have shown that in their theory a particular conformal frame can be chosen in the case of the smooth fluid approximation equations so that $G^* = \Omega^2 G$ is a constant and that in this frame the Einstein field equations (with zero cosmological constant) result. For equations (38) there are two special conformal frames in which, after the mapping (43) is made,

$$G^* = \Omega^2 G = \alpha\{\overset{*}{m}_1{}^2 - \overset{*}{m}_2{}^2\}^{-1} \tag{45}$$

is constant. These frames will be called I and II.
Frame I. In this frame

$$m_1{}^* = \text{constant} \times m_2{}^* = \text{constant} \tag{46}$$

a result which by (43) can only occur when

$$m_1 = \text{constant} \times m_2 \tag{47}$$

though the m_ν need not be constant. Thus G^* is constant and the field equations (38) map into the Einstein field equations

$$R_{ij}{}^* - \tfrac{1}{2}g_{ij}{}^*R^* = -KT_{ij}{}^* \tag{48}$$

where

$$K = 6\lambda G^*/\alpha. \tag{49}$$

Frame II. If (46) does not hold, Ω can be chosen as

$$\Omega^2 \propto m_1^2 - m_2^2 \tag{50}$$

such that

$$G^* = \alpha\{m_1{}^{*2} - m_2{}^{*2}\}^{-1} = \text{constant}. \tag{51}$$

For the remainder of this section the stars will be dropped. It follows from (51) that

$$m_1 m_{1;i} = m_2 m_{2;i} \tag{52}$$

and

$$m_{1;i} m_{1;j} + m_1 m_{1;ij} = m_{2;i} m_{2;j} + m_2 m_{2;ij}. \tag{53}$$

Equations (38) then become

$$(\alpha G^{-1}/6)(R_{ij} - \tfrac{1}{2} g_{ij} R) = -\lambda T_{ij} + \sum_{\nu=1,2} (-1)^{\nu+1}\{m_{\nu;i} m_{\nu;j} - \tfrac{1}{2} g_{ij} m_{\nu;k} m_\nu{}^{;k}\} \tag{54}$$

or

$$R_{ij} - \tfrac{1}{2} g_{ij} R = -K(T_{ij} + S_{ij}) \tag{55}$$

where

$$K = 6\lambda G/\alpha \tag{56}$$

$$S_{ij} = (\alpha/\lambda G)(\theta_{;i}\theta_{;j} - \tfrac{1}{2} g_{ij} \theta_{;k}\theta^{;k}) \tag{57}$$

and

$$\theta = \ln A(m_1 + m_2) \tag{58}$$

A being constant. The covariant derivative of (55, 57) gives

$$T^{ij}{}_{;j} = -S_{;j}{}^{ij} = -(\alpha/\lambda G)\theta^i \Box \theta. \tag{59}$$

The field equations (55) are the scalar-tensor equations used in the Dicke version of the Brans–Dicke theory (Brans and Dicke 1961, Dicke 1962), in the HN field theory (Hoyle and Narlikar 1963) and in similar theories.

In Dicke's theory, it is also required that

$$T_{;j}{}^{ij} = -\tfrac{1}{2} M \theta^i T \qquad M = 2(1 + 2\omega/3)^{-1/2} \tag{60}$$

Dicke uses the scalar λ where

$$\ln \lambda = M\theta \tag{61}$$

$$S_{ij} = \frac{(6/KM^2)(\lambda_{;i}\lambda_{;j} - \tfrac{1}{2} g_{ij} \lambda_{;k}\lambda^{;k})}{\lambda^2}. \tag{62}$$

To enable (55, 57) to include the HN field theory, it is necessary to define

$$C = i\theta = \mathrm{i} \ln A(m_1 + m_2) \tag{63}$$

so that

$$S_{ij} = -f(C_{;i}C_{;j} - \tfrac{1}{2} g_{ij} C_{;k} C^{;k}) \qquad f = \alpha/\lambda G. \tag{64}$$

The i in (63) is required to obtain the scalar tensor S_{ij} with negative energy density. In their field theory it is also required that

$$T^{ij}{}_{;j} = f C^i \Box C = C^i j^k{}_{;k} \tag{65}$$

where $j^k = \rho u^k$ is the mass current.

Thus the interesting result is obtained that the field equations (38) map into either the Einstein equations or into the scalar-tensor equations (55, 57) such that in both cases after the mapping is made, G is a constant.

8. The sign of G in theory IA

In the HN particle theory, Hoyle and Narlikar (1966, 1967) and Islam (1967, 1968) examine the geometry in the neighbourhood of one particle. Islam (1970) extends this work and shows that there exists a static solution for two isolated particles. The metric for this solution takes different (though always axially symmetric) forms for different special conformal frames. In the two frames which Islam discusses, there exists a two dimensional hypersurface enclosing the two particles such that G is positive outside, negative inside (in the neighbourhood of the particles) and tending to $\pm\ \infty$ on the hypersurface. He also shows that at least one of the curvature invariants is infinite on the hypersurface so that the singularity is not removable by a coordinate change.

Islam's work can be carried over into theory IA, even in the case when one of the isolated particles contributes positive inertia and the other contributes negative inertia. As the details are very similar to those in Islam's paper, they will not be reproduced here. A static solution for the two particles does exist and if, say, the particles are in a region where G is positive, there exists a two dimensional hypersurface surrounding the particle which contributes positive inertia such that G tends to $\pm\ \infty$ on this hypersurface and is negative inside. G is positive, however, in the neighbourhood of the other particle.

The physical nature of the hypersurfaces in Islam's case and in this case have not yet been investigated.

On the macroscopic scale, the theory IA has regions where G is positive and other regions where G is negative. As intermediate hypersurfaces are approached, $G \to \pm\ \infty$. The nature of such hypersurfaces also remains to be investigated. No similar hypersurfaces exist in the HN theory on the macroscopic scale.

9. Discussion

After HN's definition of $m^{(a)}$ in (2) is generalized by the definition (12), two forms of a generalized action, J_{I} and J_{II}, are made and two particle theories then developed. The definition J_{II} in (15–II) follows the same lines as HN's theory of electrodynamics, their suggested action (11) and Hawking's suggested action (10). Thus this generalization may seem to some readers the only correct generalization.

At various points in the papers by HN and by Islam on the particle gravitational theory, the authors refer to the case when the negative sign in the definition of (2) is replaced by a positive sign and then discuss the resultant effects of this substitution. The action however is kept in the same form. For example, Islam (1968) defines

$$m^{(a)} = -\sigma \int \tilde{G}\,\mathrm{d}a \qquad \sigma = \pm 1. \tag{66}$$

It was with this in mind that the theory of paper I was developed. This was changed slightly to theory I derived from the action J_{I} of (15–I).

The most interesting case in theory I is when both particles contributing positive inertia and those contributing negative inertia are included and the definition (20) for the λ is made. The resultant particle equations have the same mathematical appearance as those in paper I with $\lambda_1 = \lambda_2$ but their development is slightly different. When the smooth fluid approximation (33, 34) is made of these equations, the resultant field equations have the interesting property that special conformal frames can be chosen so that (35) map either into Einstein's field equations if (47) holds or otherwise into the scalar–tensor equations of the HN or Dicke field theories. Thus the equations (35) play an important mathematical role in that they are the conformally invariant version of both the Einstein and the scalar–tensor field equations.

λ_a in theory I does not have a direct physical interpretation except as a coupling factor associated with particle a. In the special case of theory IA, λ is the same as the HN coupling constant in (2). In theory II, λ_a can be interpreted as a weight factor associated with a or as the gravitational charge of a.

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